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Catalogue of 20457 Star Positions Obtained by Photography in the Declination Zone -480 to -540 (1950)

by

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ABSTRACT

The methods for constructing a catalogue of 20457 star positions for the epoch 1964 between -48° and -54° declination are described. The positions were obtained by the overlap method, and images generated by a coarse diffraction grating were employed to control magnitude related effects on the positions. A selection of about two faint AC stars per square degree was included to serve as material for the eventual determination of magnitude effects on the AC positions. The standard error of a catalogued position estimate based on two images is 0.12 in either coordinate.

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I. INTRODUCTION

There is a conspicuous gap between -50° to -52° in the photographic AGtype zone catalogues in the southern hemisphere. This is due to the lack of
coverage between -50° to -60° by the Yale Photographic Catalogue and between
-40° and -52° by the Cape Photographic Catalogue (cf. Eichhorn, 1974, pp. 246
and 274). The Cape astronomers apparently did not construct another new
independent catalogue between -40° and -52° because this region had been
repeatedly covered at the Cape before. (Gill and Hough, 1923, STS 10012);
Spencer Jones and Jackson, 1936; Spencer Jones and Jackson, 1939, STS 1003.)
No reasons were given to omit the zone between -50° and -60° from the Yale
catalogue. In view of the existence of the above mentioned coverage of the
zone -40° to -52°, it is not really surprising that initially, new
photographic observations were planned for the computation of precise
positions and proper motions in those declination belts where there were none
available at all.

The primary aim of the present catalogue is to close this gap with a generous overlap on both sides. At the beginning of the nineteen sixties, when the work on the present catalogue was planned, the authors felt that the availability of modern measuring machines and then even more so, computers, had ushered in a new era of relative photographic catalogue astrometry, since the calculation of the stars' final positions from the measured coordinates of their images on the plates could now be performed on computers. Not only did the computers eliminate the tedium of calculating the final catalogued positions "by hand", but they also freed the catalogue astrometrists from the

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²⁾Here and in the rest of this paper, STS refers to the number assigned to the catalogue in question by Sevarlic, Teleki and Szádeczky-Kardoss (1978) in their list.

necessity to find computational shortcuts, often at the cost of accuracy. Without these shortcuts, the massive computations necessary for the calculation of the final coordinates could not have been performed at all. Computers made it possible for those who designed the procedures to be employed for the computation of catalogues to turn their efforts to extracting position estimates of the highest possible accuracy and precision from the "observations", that is, the measured rectangular coordinates of the stare images on the plates.

The first step toward this goal was the introduction of the plate overlap method (Eichhorn, 1960). This method for constructing photographic star catalogues attacks the problem directly. Since the computing task of constructing a photographic star catalogue consists of calculating positions (strictly speaking, position) estimates) from the measured coordinates (the observations) and previously available position estimates of a sample of stars (the catalogued positions of the reference stars), the overlap method sets up the equations rigorously in such a way that the positions of all stars under consideration, together with the (really not very interesting) plate parameters, appear in them as unknowns. This approach avoids the somewhat paradoxical situation of the classical plate constant method in which the ultimately published postions are obtained as the (weighted) means of the several different "best" positions which result from the individual and mutually indepdent reduction of the measurements on all those plates on which any particular star occurs. After all, a "best" position which is later overridden by an even better one could not have been the best to begin with. The overlap method requires a computing effort of typically two orders of magnitude greater than does the classical plate constant method in which each plate is reduced as a single unit independently of any other plate, even

though other plates may cover the same region of the sky. Numerical effort is, however, (within reason) not a significant consideration in this age of computers, and the reduction of star catalogues by the overlap method may today be regarded as the accepted standard.

This was not yet so when we constructed this catalogue. We had tested the plate overlap method previously by re-reducing the measurements which had been made for the establishment of the Cape Photographic Catalogue -52° to -56° and -56° to -60° ; (Lukac et al., 1970) and found it to give significantly improved results, cf. Eichhorn (1974, pp 277-278).

We therefore applied it to compute the position estimates in the present catalogue, which is intended to cover part of the gap left by the Yale and the Cape Photographic Catalogues.

The present catalogue contains position estimates not only for stars to the magnitude limit one expects to find in a traditional AG type catalogue, but also for a selection of faint stars from the corresponding zones of the AC. (This is also true for the Yale zone -70° to -90°; Lü, 1971, STS 2010). The rationale for deriving estimates for the positions of these faint stars was given by Eichhorn (1974; pp 286-288); their purpose is to serve eventually as material for the assessment of the magnitude-only-dependent systematic errors (model deficiencies) in the AC zones. In order to make the catalogued positions of faint stars suitable for their intended purposes, we had to pay close attention to avoiding magnitude dependent errors, and our efforts toward this goal will be described below in Sec. 5.

II. THE PLATES

The Plates were exposed on the Taylor, Taylor and Hobson catalogue camera of the Sydney Observatory. They cover an area of 6° x 6° with a scale of

116"1/mm. The particulars were extracted from a letter of the then Government Windy
Astronomer, Harley Wood, to the Commanding Officer of the U.S. Map Service,
dated July 20, 1964. Each Ilford Rapid Process Experimental Emulsion plate
was exposed for six minutes and developed in a tropical developer which,
according to the experience of the Sydney astronomers, minimized emulsion
shift. A coarse diffraction grating (grating constant of 349) was used.

The plates were taken with the telescope always west of the pier in 1964, from February 13 to October 29, and thus may be regarded as virtually contemporaneous. Only those plates on which stars of tenth magnitude produced clearly measureable images were forwarded for measuring.

The declinations of the plate centers were always -51° (1950). Table I below gives their right ascensions (1950) and the dates and epochs of exposure. The observer is indicated in the next column (R: W. H. Robertson, S: K. P. Sims and W: H. W. Wood). The remaining columns list the hour angle at midexposure and the seeing on an arbitrary scale of 5 (best) to 1 (worst still permitting acceptable plates to be taken). The last column gives the plate numbers in the Sydney archives.

III. THE MEASUREMENT

All stars <u>selected for measurement</u> in the zone covered by the plates are listed in at least one of the following: The Catalogue by Gill and Hough (1923), and the zone -52° to -56° of the Cape Photographic Catalogue (CPC); (Jackson and Stoy, 1955; STS 1702). Furthermore, added to the list of program stars were approximately two of the faintest stars (per square degree) from the appropriate Astrographic Catalogue (AC) zones (Cape and Sydney).

As already stated above, these latter are intended to provide material for the eventual determination in the AC zones of those aberrations (that is,

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TABLE I. The plates.

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R.A.	Date	Epoch	Obs.	h.a.	S	No.	R.A.	Date	Epoch	Obs.	h.a.	S	Х
!	1964	1964					_	1964	1964				
0µ08m	Aug. 6	.60	R	2^{m} 0	3	4248	12 ^h 08	m Mar.17		W	0.00	2	40
24	Aug. 6		R	3.0	3	4249	24			W	1.5	ī	40
40	Aug. 6		R		3	4250	40			W	1.0	ĩ	40
56	Aug. 6		R	4.0	3	4251	56			W	4.0	ĭ	40
1 12	Aug. 6		R	2.5	3	4252	13 12			W	3.5	ĭ	40 40
28	Aug. 6		R	4.0	3	4253	13 28	Mar.17	. 21	W	-6.5	0	40
44	Oct.27	.82	W	0.2	2	4309	13 44	Mar.18	.21	W	-1.5	2	40
2 00	Oct.28	.82	W	-0.2	2	4315	14 00	Mar.18	.21	W	-0.5	3	40
16	Oct.28	.82	W	1.3	2	4316	16	Mar.18	.21	W	1.5	3	40
32	Oct.29	.83	W	1.4	2	4317	32	Mar. 18	. 21	W	3.0	3	40
2 48	Oct.29		W	1.0	2	4318	48		. 21	W	5.5	3	40
3 04	Oct.29		W	1.8	2	4319	15 04			S	7.5	1.2	41
20	Oct.29		W	2.9	2	4320	20	Jul.13	.53	S	6.5	1.5	41
36	Oct.29		W	5.0	2	4321	36			S	5.5	1.5	41
52	Oct.29	.83	W	5.3	2	4322	52	2 Jul.13	.53	S	4.0	1.5	41
4 08	Oct.29		W	5.6	2	4323	16 08			S	2.5	2	41
24	Oct.29		W	4.9	2	4324	24			S	6.5	2	41
40	Oct.28		W	0.6	2	4310	40			R	7.5	2	41
56	Oct.28		W	0.2	3	4311	56			R	2.5	2	42
5 12	Oct.28	.82	W	2.1	2	4312	17 12	Z Jul.12	.53	R	1.5	3	41
5 28	Oct.28		W	1.4	2	4313	28			R	0.0	3	41
44	Feb.13		W	-0.5	3	4002	44			R	0.0	3	41 41
6 00	Feb.13		W	0.0	3	4003	18 00			R	1.5	2.5	41
16	Feb. 13		W	1.5	3	4004	16			R	1.5	2.5	41
32	Feb.13	.12	W	1.0	4	4005	32	2 Jul.30	.58	W	0.0	3	42
48	Feb. 13	.12	W	0.0	4	4006	18 48	Jul.30	.58	W	3.0	3	42
7 04	Feb.13		W	6.0	4	4007	19 04			W	4.5	3	42
20	Mar. 4		W	0.0	3	4011	20			R	2.0	3	41
36	Mar. 4		W	0.0	3	4012	36			R	2.0	3	41
52	Mar. 9		R	4.0	2	4013	52			R	2.0	3	41
8 08	Mar. 9		R	2.0	2	4014	20 08			- R	6.0	2	41
24	Mar. 9		R	1.5	2	4015	24			R	2.5	2	411
40	Mar. 9		R	2.0	2	4016	40			R	2.0	3	41
56	Mar. 9		R	0.5	2	4017	56			R	1.0	3	42
9 12	Mar.11	.19	R	1.5	2	4028	21 12	Jul.15	.54	R	2.0	3	41 42 42
28	Mar. 9		S	-0.5	2	4018	21 28			R	11.0	3	42
44	Mar. 9		S	1.0	2	4019	44			R	8.0	3	42
0 00	Mar. 9		S	1.0	2	4020	22 00			R	3.0	2	42
16 32	Mar. 9		S S	0.5 1.0	$\begin{smallmatrix}2\\2.5\end{smallmatrix}$	4021 4022	16 32			R R	4.0 3.0	2 2	42
,													- 1
0 48	Mar. 9		S	1.0	3	4023	48			R	4.0	2	49
1 04	Mar. 9		S	1.0	3	4024	23 04			R	3.5	2	4.5
20	Mar.10		S	1.0	3	4025	20			R	4.0	3	4
36 52	Mar.10		S S	1.0	3 3	4026	36 5.4	_		R	4.0	3	4.5
32	Mar.10	.19	Ŋ	3.0	J	4027	54	Aug. 6	.60	R	2.5	S	44

deviations from strictly gnomonic projection) that depend on magnitude alone. We employed coarse objective grating images in the computation of the present catalogue to determine and eliminate magnitude dependent aberrations with the hope of having minimized their effect on the published positions. It can be seen that magnitude dependent aberrations which do not depend also on the position of the image on the plate can be determined in only two ways. The first of these requires stated during one single exposure, some device which will produce on the plate two well separated (but still very close) images of each star such that the light striking the objective is split into two unequally intense parts, both of which will form images of different blackening. Examples are the coarse objective grating and the Fellgett prism. The second possibility for determining magnitude-only-dependent aberration is by comparison with such reference material which is presumably free from such errors. Since no device as described in (a) was ever employed for taking any of the plates later measured for the construction of the AC, those aberrations in the AC whose effects depend on magnitude alone cannot be determined in any other way than by comparison with reference material. It is clear that the magnitude range of this reference material must encompass that of AC stars, because the magnitude-only-dependent aberrations are most likely nonlinear functions of the magnitudes. To have included faint AC stars in the present catalogue is therefore the first step for the establishment of the required reference material. The second and last step toward this aim will have been accomplished with the determination of the proper motions of these stars. These proper motions will be necessary for the computation of positions at the epochs of the AC plates.

The <u>faint AC stars</u> were selected by looking for listed coordinates of appropriately faint stars close to the centers of the AC plate quadrants.

From the spherical coordinates of the tangential points of the plates and the focal length of the instrument on which the plates were exposed, the expected rectangular coordinates of the program stars were predicted by the precepts given by Eichhorn (1974, pp. 292-295).

The machine on which these plates were measured is a Mann 422/F with binocular eyepiece and manually operated lead screws on which both rectangular coordinates are measured simultaneously. Each screw is connected to an encoder with optical readout (Mann Data Logger, with a least count of one micrometer). During the measurement sittings, the correspondence between the raw screw readings (on the drums) and the encoder readings were checked eve. hour to guard against slippage between encoders and screws. The encoder readings were automatically punched on cards. Each encoder can be set such that the constant difference between its reading and that of the corresponding lead screw is arbitrary; also, each encoder can be set to increase its reading either with or counter to that of the corresponding lead screw. In combination with operating the turntable, it is thus easy to adjust a plate in the measuring machine in such a way that the encoders duplicate the predicted coordinates of the stars' images within better than ten micrometers. This almost eliminates identification errors during measurement. Because of the features described above, the measured coordinates of an image in the reverse position can likewise generally be made to agree within a few micrometers with the coordinates of the same image that were measured in the direct position. This agreement of measured coordinates in direct and reverse positions allows the measurer to know immediately whether the image measured was the one he intended to measure.

In order to minimize the influence of systematic and measuring machine generated errors (such as screw errors and way errors) on the measured

coordinates, the initial position of a plate during the measurement in the direct position was turned by 90° against that used for the plate immediately preceding in right ascension. Since the spherical coordinates published in this catalogue typically depend on the coordinates measured on two plates, most of the right ascensions as well as the declinations were thus computed from measurements made with both screws.

Before the measurements started, the screws and ways of the measuring machine had been investigated and found to impart no systematic errors larger than one micrometer on the measurements. Corrections for such errors were therefore not applied.

The <u>measuring team</u>, consisting of seven measurers, was under the direct supervision of W. J. Veigel.

The plates being measured lay emulsion up on a horizontal stage which consists in part of a thick transparent clear glass pane. During the measurements, the position of the plate being measured, with respect to the stage, was defined by three (vertical) cylindrical studs against which the plate was held by springs. Also, during measurement the plate was pressed down against the glass pane by springs. The studs defined the position of the plate so well that it could be completely taken off the stage and be reinserted within about 5 micrometers of its original positions if nothing else were changed. On the average, more than 500 images per plate were measured and more often than not, the measurements of the images in one position were spread over two or three separate sittings. Only by removing the plates from the stage between sittings could we make sure that all measurements on any one plate were made by the same measurer. We judged this preferable to having a plate measured by different measurers and thus running the risk of introducing observer-dependent systematic errors. It was

therefore unnecessary to investigate systematic differences between different measurers. In direct and reverse position of the plate, a "measurement" on an image consisted of the average of the readings made at three settings each. The readings during the reverse measurements were punched onto the same cards as those made during the direct ones. All ninety plates were measured between December 1966 and October 1967.

In order to "homogenize" the measurements, that is, to find those coordinates which would have been measured had the plates not been removed from the measuring machine between sittings, and had they not shifted with respect to the screws during the sittings, nine control stars were selected on each plate - one close to the center of the plate, one close to each of the corners and one more close to the middle of each edge. These control stars were measured at the beginning and the end of each sitting.

The <u>homogenization of the measurements</u> of the coordinates on one plate from the different sittings (both in direct and reverse positions) was accomplished as follows.

Let \underline{x}_{ij} , \underline{y}_{ij} , be the rectangular coordinates of the <u>i</u>-th (control or field) star measured during the <u>j</u>-th sitting. Since the temperatures at the measuring machine were kept sufficiently constant so that a relative expansion of the plates with respect to the measuring screws need not be allowed for, we may postulate

$$x_{i1} = x_{ij} \cos \varphi_j + y_{ij} \sin \varphi_j + C_j$$

$$y_{i1} = -x_{ij} \sin \varphi_j + y_{ij} \cos \varphi_j + D_j$$
(1)

where \P_j , Q_j , and D_j are constants for each sitting. Equations (1) are always available for the control stars in each sitting. For the coordinates of all stars (field and control), measured during sittings j and k, we have the equations of condition.

 $\underline{x_{ij}}\cos\varphi_j + \underline{y_{ij}}\sin\varphi_j + \underline{C_j} - \underline{x_{ik}}\cos\varphi_k - \underline{y_{ik}}\sin\varphi_k - \underline{C_k} = 0$ $-\underline{x_{ij}}\sin\varphi_j + \underline{y_{ij}}\cos\varphi_j + \underline{D_j} + \underline{x_{ik}}\sin\varphi_k - \underline{y_{ik}}\cos\varphi_k - \underline{D_k} = 0$ available for the constants of the j-th and the k-th sitting. Normally, except for the control stars which were meatured during each sitting, the plate was in the direct position during one of the sittings j and/or k and in the reverse position during the other one. The solution of the normal equations which result from the condition equations (1) and (2) is easily calculated on a computer. In these normal equations, $\underline{x_{ij}}$ and $\underline{y_{ij}}$ are the observations, and the sitting constants φ_j , $\underline{C_j}$, $\underline{D_j}$ are the adjustment parameters.

Generally, more stars per sitting were measured during sittings in the reverse plate position than with the plate in the direct one. In the reverse position, all stars on any one plate were quite often measured during one sitting, especially when the star density on the plate was not particularly high. This is so because the finding list for the reverse position measurements was a printout of the measurements made in the direct position, while the finding list for the measurements in direct position was a computer sheet prepared at the Defense Mapping Agency (DMA, then Am, the Army Map Service). The reverse sittings thus usually overlapped the direct ones, thereby generating a structure of equations of the type (2) which greatly contributed to the accurate determination of the sitting constants. These constants were then used to transform, by equations of the type (1), all measurements to what they would have been had they been made during the first sittings. This evidently comprises the taking of the means of the direct and reverse measurements, and regarding these means as the measured rectangular image coordinates in all further computations.

The precision of each individual measurer was constantly monitored

without his knowledge by comparing the direct and reverse positions, each derived from one set of measurements, after the homogenization. On the average, the dispersion of the differences between the sets of direct and reverse measurements on the same star is 1.09 micrometers, with the figure for the best measurer being 0.89 micrometers, and 1.59 micrometers for the worst.

THE

IV. THE REFERENCE POSITIONS AND CATALOGUE SYSTEM

The raw material for the calculation of the reference positions which were used to reduce the plates was taken from these sources:

- 1. 6744 Estrellas del Catalogo de Boss (Martinez, 1959); LaPl30, STS

 1775, which gives observations of star positions in the region -47°

 to -82° made at the La Plata Observatory. The average epoch of these positions is 1945 and the system is ostensibly that of the FK3,
- 2. The GC, (STS 1727) and
- 3. The Third Cape Catalogue for 1950.0 (Stoy, 1966); Cp₅₀PP, STS 1697, which lists the positions of 2805 stars on the system of the FK4 for an epoch around 1964.

LaPl30 contains independent new positions of CC stars. Our original intention had been to use these for revising the CC positions and proper motions in the way described by Eichhorn and Googe (1969). In order to accomplish this, we reduced the LaPl30 positions onto the FK4 system by applying the tables (FK4-FK3) published in the FK4, and reduced the CC positions to the system of the FK4 using the tables by Brosche, Nowacki, and Strobel (1964). Only the stars in the region from -47°30' to -54°30' were included. After this had been done, considerable systematic differences between the positions of the various stars from different sources were still conspicuous. We therefore decided to disregard Martinez' statement that

LaPl30 is on the FK3 system and to investigate the systematic difference between the LaPl30 and the GC with the aim of deriving new tables for the reduction of the system of the LaPl30 to that of the GC. The same is also true, - mutatis mutandis - for Cp₅₀PP.

After computing the GC positions for the epoch of the LaPl30 positions, the differences GC minus LaPl30 were investigated first. For this purpose, the star positions were grouped according to right ascension (by hours), declination (less or greater than -51°) and magnitude (less or greater than 7429, which is the median magnitude of the 1868 stars in this sample). The averages of the right ascension and declination differences were then calculated for each subset together with the dispersions about the averages. Inspection of these differences, given in Table II, reveals their dependence on right ascension, declination and magnitude. In particular, a bias in right ascension is apparent.

We decided to represent the systematic differences by an analytic formula. Our methods were modeled somewhat on those of Brosche (1966), who expands the systematic differences between the FK4 and CC in terms of spherical harmonics which are first orthogonalized by the Gram-Schmidt process with respect to the data at hand, namely the stars common to the two catalogues. He then computes the coefficients and rejects (that is, sets equal to zero) those which do not pass a significance test based on the Fischer distribution. Since the functions, that is, the spherical harmonics themselves are orthogonal, elimination of the ones found to make an insignificant contribution to the analytical representation of the difference will not influence the values of the coefficients of those retained.

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Δδ στ Π Δακωδ σ ₅ Π Δακωδ σ ₅ Π Δακωδ σ ₅ Π Δακωδ σ ₅ Π σ ₇ Π σ ₇ Π σ ₇ Π σ ₇	<-51 ⁰	60	· CO	۷I	510			S Co	c -51 ₀				5- VI	10		•	Alleria / Comments of
.4 .5 13 ±.5 .8 .5 .7 29 7 .5 .4 .9 .4 .6 13 ±.5 .7 .2 1.0 22 7 .5 .4 .9 .4 .8 13 ±.4 .9 .2 1.0 22 7 .5 .4 .9 .4 .7 7 7 .4 .7 1.0 .2 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 .9 <th>DS of II Bacos</th> <th>3</th> <th>3</th> <th>6.</th> <th>37</th> <th>દ</th> <th>=(</th> <th>Jacos</th> <th>64</th> <th>60</th> <th>б</th> <th>= </th> <th>Dacoos</th> <th></th> <th>90</th> <th>8</th> <th>=1</th>	DS of II Bacos	3	3	6.	37	દ	= (Jacos	64	60	б	=	Dacoos		90	8	=1
.4 .6 13 +.5 .7 .2 1.0 22 7 .5 .4 .9 .4 .8 13 +.8 .9 .2 .8 21 9 .7 .3 .9 .4 .7 .7 .9 .2 .8 21 7 .9 .7 .3 .4 .7 .11 .4 .9 .2 .0 .2 .9 .2 .9 .2 .9 <td< td=""><th></th><td></td><td>œ</td><td>∞.</td><td>4.</td><td>S</td><td>13</td><td>ા. જ</td><td>∞.</td><td></td><td>7.</td><td>29</td><td>7</td><td>1.0</td><td>ر. دن</td><td>∞.</td><td>17</td></td<>			œ	∞.	4.	S	13	ા. જ	∞.		7.	29	7	1.0	ر. دن	∞.	17
5 .2 3	∞	9	9.	2	4.	9.	13	٠,	.7		1.0	22	7	.5	4.	6.	14
.4 .8 13 4 .9 .2 1.0 22 7 .9 .0 1.3 .4 .7 .7 .7 .7 .9 .4 .7 26 7 .10 .2 .9 .4 .7 .1 .2 .8 .2 7 .9 .9 .9 .5 .6 .3 7 .1 .8 .2 7 .9 .0 .1 .2 .6 .3 7 .1 .8 .2 .9 .1 .7 .7 .9 .0 .1 .7 .1 .8 .7 .1 .8 .1 .7 .3 .7 .1 .8 .1 .7 .1 .8 .1 .7 .1 .8 .1 .7 .1 .6 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .2 .1 .2 .1 .1 .2 .2 .2 .1 .1 .2<	14	2.	~	.5	٠.	.2	က		6.		∞.	21	6.	۲.	. .	6.	12
.4 .7 7 .4 .7 7 .4 .7 11 .4 .7 11 .4 .7 24 .4 .7 24 .5 .6 33 -2 .2 .6 33 -2 .9 .2 .6 34 -7 .9 .9 .9 .2 .6 33 -7 .9	2	4-	4.	∞.	4.	φ.	13		o.		1.0	22	7	6.	0.	1.3	14
.4 .7 11 -3 .9 .2 .8 23 -1.1 .8 -2 .9 <	11	9.	9	. 2	4.	.7	2		6.		.7	5 6	2.7	1.0	.2	6.	18
.4 .7 24 ±.3 .9 .3 .8 22 3 .8 .7 .7 .7 .7 .7 .7 .7 .7 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .6 .7 .9 .1 .9 .2 .9 .1 .9 .1 .9 .1 .9 .1 .1 .9 .2 .2 .9 .1 .1 <	17			4.	4.	.7	11		٠.		∞.	23	-1:1	∞.	2	6.	16
.5 .6 33 2 .9 .2 1.1 .2 .6 37 4 .6 .1 .8 33 7 .9 .1 .6 .1 .6 .26 4 .6 .1 .7 .4 .6 .1 .8 .1 .6 .2 .7 .1 .8 .1 .6 .2 .7 .1 .8 .1 .6 .7 .1 .8 .1 .6 .7 .1 .8 .1 .6 .8 .1 .6 .13 6 .1 .0 .8 .1 .7 .1 .8 .1 .7 .1 .1 .7 .1 .1 .7 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .2 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .1 .2 .2 .2 .2 .2 .2 .2 .2 .2 .2 .2 .2	24			2	۲.	.7	24		c.		∞.	22	٠.	∞.	.7	.7	12
.2 .6 37 ±.9 1.0 .1 .8 33 7 .9 .1 .6 .1 .6 26 ±.4 .5 .1 .7 24 5 .7 .1 .8 .1 .6 26 ±.4 .6 .3 .6 .3 .6 .9 .7 .1 .8 .9 .2 .8 .2 .8 27 6 .9 .3 .6 .13 8 .9 .2 .8 .9 .2 .8 .9 .2 .8 .9 .1 .7 .1 .9 .1 .9 .4 .7 .1 .9 .1 .0 .9 .4 .7 .1 .8 .1 .7 .1 .9 .8 .8 .8 .8 .8 .9 .1 .1 .9 .1 .1 .9 .1 .0 .9 .1 .1 .9 .9 .2 .2 .2 .2 .2 .2 .2 .2 .2 .2 <th>33</th> <td></td> <td></td> <td><u>~</u></td> <td>ĸ.</td> <td>9.</td> <td>33</td> <td></td> <td>1.0</td> <td></td> <td>9.</td> <td>31</td> <td>2</td> <td><u>ئ</u></td> <td>.2</td> <td>1.1</td> <td></td>	33			<u>~</u>	ĸ.	9.	33		1.0		9.	31	2	<u>ئ</u>	.2	1.1	
.2 .5 24 6 .9 .1 .7 24 5 .7 1 .8 .1 .6 26 9 .1 .7 34 5 .7 .1 .6 .1 .6 20 9 .8 1 .5 26 9 .8 .1 .7 .1 .8 .9 .2 .8 .2 .8 .27 6 .9 .3 .6 .11 .9 .1 .7 .3 .4 .7 .9 .4 .7 .10 4 .9 .6 .8 .5 .9 .2 -1 .0 .9 .4 .7 .10 4 .9 .6 .8 .6 .6 .4 .7 .10 4 .9 .6 .8 .7 .9 .2 .7 .7 .9 .6 .9 .4 .10 .9 .4 .10 .9 .4 .7 .9 .2 .9 .4	39			~	7.	9.	37		1.0		∞.	33	7	e.		9.	44
.1 .6 26 26 34 5 .7 .1 .6 .1 .6 20 20 20 26 5 1.0 .8 .2 .5 26 .4 .6 13 8 .9 .2 .8 .2 .8 .2 6 .9 .3 .6 19 -1.1 .9 .1 .7 .3 .8 .7 .9 .1 .8 13 6 .1 .7 .1 .9 .1 .0 .3 .6 .23 7 .7 .9 .4 .7 .1 .8 .0 .1 .0 .3 .8 .7 .0 .2 .7 .7 .9 .6 .8 .5 .9 .2 .7 .0 .4 .9 .6 .8 .6 .8 .5 .9 .4 .7 .0 .4 .9 .6 .8 .6 .8 .9 .6 .8 <t< td=""><th>29</th><td></td><td></td><td>~</td><td>~.</td><td><u>د:</u></td><td>2.4</td><td></td><td>Ç.</td><td></td><td>7.</td><td>24</td><td>٠. ئ</td><td>.7</td><td></td><td>∝.</td><td>97</td></t<>	29			~	~.	<u>د:</u>	2.4		Ç.		7.	24	٠. ئ	.7		∝.	97
.1 .6 20 9 .8 1 .5 26 5 1.0 0 .8 .2 .5 26 .9 .9 .6 13 8 .9 .2 .8 .3 .7 .8 .7 .6 .13 9 .1 .7 .5 .3 .2 .9 .1 .8 13 6 1.2 1 1.0 .5 .3 .2 .9 .4 .7 .1 .8 .0 1.0 .5 .9 .2 .7 .7 .10 4 .9 .6 .8 .6 .6 .4 .7 .10 4 .9 .6 .8 .7 .9 .2 .7 .1 .9 .4 .9 .6 .8 .6 .6 .4 .9 .6 .4 .9 .6 .9 .4 .9 .6 .9 .4 .9 .6 .9 .4 .9 .6 .9	7.7	7.		•	-:	œ.	5.6		≈ .		."	ž	٠. د		₹.	œ.	%
.2 .5 26 9 .6 13 8 .9 .2 .8 .2 .8 27 6 .9 .3 .6 19 -1.1 .9 .1 .7 .3 .7 18 6 .9 .1 .8 13 6 1.2 .1 1.0 .5 .8 .28 7 .9 .4 .7 10 .9 .1 .9 .1 .9 .6 .8 .5 .9 .2 -1.0 .9 .4 .7 10 4 .9 .6 .8 .6 .6 .14 .1 1.4 0 .6 21 4 1.3 .2 .6 .8 .6 .6 .14 .1 1.4 0 .6 21 4 1.3 .2 .6 .6 .7 .7 .7 .9 .2 .5 1.7 5 .8 .5 1.3 .8 .5 .3 .8	.5 14	>-). 			٠:	20		≈ .		મ્:	97	ر. د	e:	٠.	×.	Σ.
.2 .8 27 6 .9 .3 .6 19 -1.1 .9 .1 .7 .3 .7 18 5 .9 .1 .8 13 6 1.2 1 1.0 .5 .8 .28 5 1.0 .0 .9 14 .1 .8 .0 1.0 .5 .9 .22 -1.0 .9 .4 .7 10 4 .9 .6 .8 .6 .6 14 .1 1.4 0 .6 21 4 1.9 .6 .8 .2 .7 7 .9 .2 .5 17 5 .9 .6 .9 .5 .3 12 4 1.0 4 .9 .6 .8 .9 .6 .8 .5 .3 12 7 1.0 3 .8 16 5 1.2 .9 .9 .9 .5 .3 12 7 .9 .2 <t< td=""><th>20</th><td>æ. 1</td><td>æ.</td><td>۰.</td><td>~.</td><td>g.</td><td>72</td><td></td><td>9.</td><td></td><td>ب</td><td>:</td><td>3.</td><td>G.</td><td>?:</td><td>≈.</td><td>-</td></t<>	20	æ. 1	æ.	۰.	~.	g.	7 2		9.		ب	:	3 .	G.	?:	≈ .	-
.3 .7 18 5 .9 .1 .8 13 6 1.2 1 1.0 .5 .8 28 5 1.0 .0 .9 14 .1 .8 .0 1.0 .5 .9 .2 1 1.0 2 .7 27 4 1.3 .3 .8 .5 .9 .2 .7 .7 10 2 .7 10 4 .9 .6 .8 .6 .6 .14 .1 1.4 0 .6 21 4 1.9 .6 .8 .2 .7 .7 .9 .2 .5 17 5 .9 .4 .9 .5 .3 12 7 .9 .2 .5 17 5 .9 .4 .8 .6 .3 8 9 .8 .4 .8 25 5 .9 .4 .8 .6 .3 8 .7 .9 .2 .5 </td <th>8.2</th> <td>9</td> <td>9</td> <td>~</td> <td>~;</td> <td>æ</td> <td>2.7</td> <td></td> <td>o:</td> <td></td> <td>ج.</td> <td>≘</td> <td>-1.1</td> <td>ੜ:</td> <td>٠-</td> <td>.7</td> <td>25.</td>	8.2	9	9	~	~;	æ	2.7		o:		ج.	≘	-1.1	ੜ:	٠-	.7	25.
.5 .8 28 5 1.0 .0 .9 14 .1 .8 .0 1.0 .3 .6 23 1 1.0 2 .7 27 4 1.3 .3 .8 .5 .9 22 -1.0 .9 .4 .7 10 4 1.3 .2 .6 .8 .6 .6 14 .1 1.4 0 .6 21 4 1.3 .2 .6 .8 .6 .6 14 .1 1.4 0 .6 21 4 1.3 .2 .6 .8 .7 .7 .7 .7 .9 .2 .5 1.7 5 .9 .4 .8 .5 .3 .8 .4 .8 .5 .7 .7 .7 .3 .8 .6 .3 .8 .4 .8 .2 5 .1 .3 .8 .6 .3 .8 .7 .9 .4 .8 </td <th>81</th> <td>9</td> <td>9</td> <td>_</td> <td>~•</td> <td>.7</td> <td>8</td> <td></td> <td>ာ.</td> <td></td> <td>æ.</td> <td>ខ</td> <td>9.</td> <td>1.2</td> <td></td> <td>1.0</td> <td>5</td>	81	9	9	_	~•	.7	8		ာ.		æ.	ខ	9.	1.2		1.0	5
.3 .6 23 1 1.0 2 .7 27 4 1.3 .3 .8 .5 .9 .2 -1.0 .9 .4 .7 10 4 .9 .6 .8 .3 .8 27 0 1.1 0 .4 .9 .6 .8 .9 .6 .8 .6 .6 .6 .1 1.4 0 .6 21 4 1.3 .2 .6 .8 .7 .7 .7 .9 .6 .2 .5 1.2 .2 .9 .9 .9 .9 .9 .9 .4 .8 .5 1.3 .8 .6 .9 .4 .8 .5 1.3 .8 .6 .9 .4 .8 .5 .9 .4 .8 .5 .7 .7 .7 .9 .8 .4 .8 .5 .7 .7 .7 .3 .8 .7 .7 .7 .7 .7 .3 .8 </td <th>20</th> <td>-</td> <td>-</td> <td>င္း</td> <td>c;</td> <td>ಪ.</td> <td>2.8</td> <td></td> <td>0.1</td> <td></td> <td>≎:</td> <td>7</td> <td>٦.</td> <td>∝.</td> <td>=</td> <td>1.0</td> <td>71</td>	20	-	-	င္း	c;	ಪ.	2.8		0.1		≎ :	7	٦.	∝.	=	1.0	71
.5 .9 .2 -1.0 .9 .4 .7 10 4 .9 .6 .8 .3 .8 27 0 1.1 0 .4 .6 4 1.3 .2 .6 .6 .6 14 .1 1.4 0 .6 21 3 1.0 4 .9 .2 .7 .7 .7 .9 .2 .5 17 5 1.3 .5 .3 12 7 .9 .2 .5 17 5 .8 .5 1.3 .6 .3 8 9 .8 .4 .8 25 5 .9 .4 .8 .3 .8 17 9 .8 .1 .8 .2 7 .7 .7 .3 .8	<u>.</u>	۲	۲.	c:	۳.	9.	23		1.0		.7	2.2	٠.4	 	ij	æ	30
.3 .8 27 0 1.1 0 .4 6 4 1.3 .2 .6 .6 .6 .1 1.4 0 .6 21 3 1.0 4 .9 .2 .7 .7 7 1.0 3 .8 16 5 1.2 .2 .9 .5 .3 12 7 .9 .2 .5 17 5 .8 .5 1.3 .6 .3 .8 9 .8 .4 .8 25 5 .9 .4 .8 .3 .8 17 9 .8 .1 .8 22 7 .7 .3 .8	17			۵.	٠,	G.	7.7		ъ.		.7	10	₽	သ့	9.	ထ္	13
.6 .6 .14 .1 1.40 .6 .213 1.04 .9 .2 .2 .7 74 1.03 .8 165 1.2 .2 .9 .5 .3 127 .9 .2 .5 175 .8 .5 1.3 .6 .3 89 .8 .4 .8 .255 .9 .4 .8 .3 .3 .8 .179 .8 .1 .8 .227 .7 .3 .8	16			0.	۳.	8.	2.2		1.1		4.	ယ	4.4	1.3	.2	ဖ	18
.2 .7 74 1.03 .8 165 1.2 .2 .9 .5 .3 127 .9 .2 .5 175 .8 .5 1.3 .6 .3 89 .8 .4 .8 255 .9 .4 .8 .3 .8 .179 .8 .1 .8 227 .7 .3 .8	193			0	9.	9.	14		1.4		9.	21	ا س	1.0	٠.4	o.	18
.3 127 .9 .2 .5 175 .8 .5 1.3 .3 89 .8 .4 .8 255 .9 .4 .8 .8 179 .8 .1 .8 227 .7 .3 .8	75			.3	.2	.7	7		1.0		∞.	16	ا ئ	1.2	.2	<u>ئ</u>	11
.3 89 .8 .4 .8 255 .9 .4 .8 .8 179 .8 .1 .8 227 .7 .3 .8	16		rs.	e.	c.	۳.	12		6.		ઙ	17	.5	∞.	٠.	1.3	14
8 179 .8 .1 .8 227 .7 .3 .8	11	4.	4	s.	9.	. .	∞		∞.		φ.	25		œ.	4.	∞.	24
	T.	1.0	0	æ	. .	∞.	17		∞.		φ.	22	L'-	2.	ო.	∞ .	15

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In place of the spherical harmonics we used products of a trigonometric polynomial in right ascension, a polynomial in magnitude and, since we are dealing with a relatively narrow declination zone, a polynomial in declination.

Among the spherical harmonics which Brosche uses for this method, there is more or less a natural order of increasing complication. (First by degree, and for harmonics of the same degree, by order.) For the functions of the variables which we adopted, no such obvious order exists, however. For instance, there is no natural way to decide which of the two is more complicated, $\sin 3\Delta$ or $\delta \cos 2\alpha$. We therefore proceeded as follows.

We defined and used functions (terms) which would be strictly orthogonal if the stars were uniformly distributed. These terms were one of the functions of the right ascension: 1, $\sin\alpha$, . . . , $\sin6\alpha$, $\cos\alpha$, . . . , $\cos6\alpha$, possibly multiplied by one from the following set: \underline{y} , $\underline{y}^2-1/3$, $\underline{y}^3-3\underline{y}/5$, and $\underline{m}-7\overline{m}5$, where $\underline{y}=(\delta+51^{\circ})/3$, with \underline{m} being stellar magnitude. Note that the functions of \underline{y} are merely unnormalized Legendre polynomials through the third degree in \underline{y} , and that \underline{y} is defined such that it varies from -1 to +1 for the declination of the stars in the catalogue. These polynomials (functions of the declination) would also be orthogonal if the stars were uniformly distributed. Similarly, the terms $\sin\underline{m}$ and $\cos\underline{m}$ are orthogonal for a uniform distribution. Note that $\underline{m}-7\overline{m}5$ is approximately orthogonal to all other terms since $7\overline{m}5$ is approximately the average magnitude.

Sixty-five different terms were tested for inclusion in the model for the systematic differences. Obviously, not all of these could be significant.

The following algorithm was thus used to arrange them in order of their effectiveness: First, that term among the sixty-five was selected which, when used by itself, resulted in the greatest reduction of the sum of the squares

of the residuals. From then on the selection proceeded inductively. Assume that n terms have been chosen. The $(\underline{n}+1)^{St}$ term is then that term from among the remaining, which, when used in conjunction with those already selected, will reduce the sum of the squares of the residuals by the largest amount. (Note however, that n terms chosen in this way will not necessarily produce the smallest sum of squares of residuals among all possible selections of n terms from the original collection of sixty-five. We do not regard this as a very important point, though.)

As this selection algorithm was applied, a selection was calculated with $\underline{n}=2,3\ldots$ terms as well as their standard errors, the sum of squares of residuals and dispersion ($\underline{R}\underline{M}\underline{S}$) of the residuals. Table III gives these data for the corrections in right ascension for $\underline{n}=2\ldots 12$ and Table IV gives similar data for declination. The first column (\underline{n}) gives the number of terms, the second (\underline{S}) the sum of the squares of the residuals in units of 1^{n2} , the third the dispersion of the residuals and the fourth, (\underline{R}), gives the ratio used in the hypothesis test for the significant term. (This will be discussed in the next paragraph.) The colæums labeled \underline{C} give the coefficients of the terms and those labeled \underline{C} give the standard errors of the \underline{C} immediately preceding it, in units of 0.001.

The <u>n</u>th term was rejected as being insignificant if in the context of the solution with <u>n</u> terms, the hypothesis that the coefficient of the last term was zero did not need to be rejected. Let $R = (\underline{S_{n-1}} - \underline{S_n})(\underline{N} - \underline{n})/\underline{S_n}$, where $\underline{S_j}$ is the sum of squares of the residuals from the solution with <u>j</u> terms, and \underline{N} is the number of equations. The hypothesis was then tested at the 95% confidence level by comparing <u>R</u> with tables of the Fisher distribution (Selby 1971, pp. 613-619). Because of the large value $\underline{N} = 1868$, a term was accepted if <u>R</u> exceeded 3.8. As can be seen from the tables, seven terms passed this

TABLE	111.	Right-a	scens i o	n-co,e f f	icients	for s	ystemati	c diffe	rences	C minu	s LaP130.
ŭ	2	3	4	5	6	7	8	9	10	11	12
<u>s</u>	1314	1308	1303	1298	1293	1290	1288	1286	1284	1282	1282
RVS	839	838	836	835	834	833	832	832	831	831	831
Ŗ	37.7	8.8	7.2	7.0	6.8	5.0	0 3.2	2.5	2.5	2.7	1.9
$\frac{C_1}{\sigma_1}$	-517 21	-514 21	-514 21	-517 21	-518 21	-520 21	-521 21	-521 21	521 21	-522 21	-521 21
$\frac{\mathbb{C}_2}{\widehat{\sigma}_2^2}$	-170	-172	-168	-167	-164	-165	-165	-164	-164	-163	-162
	30	30	30	30	30	30	30	30	30	30	30
\mathcal{C}_3		81 30	80 · 30	76 30	78 30	76 30	87 . 31	89 31	94 31	95 31	96 31
<u>C</u> ₄ C ₄			108 44	116 44	116 44	119 44	121 44	120 44	115 44	109 44	112 44
<u>C</u> 5 σ ₅				-108	-115	-114	-114	-115	-114	-113	-112
<i>σ</i> ₅				45	45	45	45	45	45	45	. 45
C 6 €6					238	237	244	238	240	247	251
					100	100	100	100	100	100	100
C?				•		62 30	61 30	50 31	47 31	43 31	44 31
						00					
C8 68							40 24	42 24	43 24	44 24	45 24
C ₉								-38	-39	-40	-39
С ₉ б ₉								26	26	26	26
Ç10									-45 31	-47 31	-47 31
10									91		
C10 10 C11 C11										-156 105	-146 105
$rac{C_{12}}{\sigma_{12}}$											130 103

	TABLE IV.	Deel in a	+ion-coef	ficients	for system	matic dif	ferences	CC minus	LaP130.
·		Deci ina	4	5	6	7	8	9	10
Ū	2		989	985	981	978	975	973	970
S	1000	994	73	73	73	73	72	72	72
RMS	73	73		;	6.9	6.1	5.3	5.0	4.4
R	11.8	10.8	10.0	7.5	••	20	20	20	20
C1 61	19 2	19 2	19 2	20 2	20	2	2	2	2
3	-5	-5	-5	-5	-5 2	-5 2	-5 2	-5 2	-5 2
C2 62	2	2	2	2		-8	-8	-8	-8
က္ကမွာ		-8 3	-8 3	-8 3	-8 3	3	3	3	3
			18	19	18	18 7	18 7	18 7	18 7
<u>C</u> ₄ σ ₄			7	7	7			6	
Ç ₅				7 3	7 3	7 3	6 3	3	6 3
5				,·	-9	-9	-10	-9	-9
<u>Ç</u> 6 6					. 4	4	4	4	4
Cn						13 6	13 6	13 6	13 6
. 57						ŭ	-8	-8	-8
0,65 Ge 0,65 Che							4	4	4
								-5 3	-6 3
Coro								•	-5
$\frac{c_{10}}{\sigma_{10}}$	0 0							•	3
C ₁	1 1								

test for inclusion in the right ascension difference formula and eleven in that for declination. The terms in α were: 1, $\cos 2\alpha$, $\sin 5\alpha$, $y\cos 3\alpha$, $y\sin 6\alpha$, $(y^3-3y/5)\sin 4\alpha$ and $\sin 3\alpha$, and in δ : 1, m-7, $\sin 3\alpha$, $(y^3-3y/5)$, $\cos \alpha$, $y\sin \alpha$, $(y^2-1/3)\cos 3\alpha$, $y\cos 2\alpha$, $\cos 5\alpha$, $\cos 6\alpha$ and $(y^2-1/3)$.

The appearance of that many complicated terms is somewhat surprising. The corrections due to these terms are, however, uniformly small. (Note that except for the term in \underline{m} -7.5, the maximum absolute value of every term is at most 1 in the range of α and δ in question). The absolute value of the effect of a term can therefore never be larger than its coefficient.

Tables III and IV deserve a few more remarks. The stability of both the coefficients and their standard errors, as more terms are added, show the effectiveness of the orthogonalization at least among the terms selected. It will also be noticed that the coefficients of the terms judged to be significant are in general at least twice their standard error but there are exceptions. This property also depends on the orthogonality of the terms.

The CC system as reduced to the FK4 system thus becomes the primary system for the whole project by adopting the flexible models for the reduction of the LaPl30 positions to it. In particular, it implies the acceptance of systematic corrections for the Cp₅₀PP reference positions (to be discussed below) as well as corrections to the early epoch positions used to derive proper motion estimates. On the other hand, if the high order terms had been rejected, the system of that final catalogue would have become a compromise. It would then not be related in any simple way to any of the components that entered into it. Thus, it would potentially be harder to relate such a system to any new consistent system of stellar coordinates that may become available

in the future (e.g., the system of the FK5). Of the three reference star sources, the ∞ is the natural choice for the primary system because it is the most numerous; it alone furnishes proper motions and it has been widely used as a standard in catalogue astrometry. The $Cp_{50}PP$ positions cover only two thirds of the zone of the present catalogue, and extrapolation would have been required if the $Cp_{50}PP$ position had been chosen to define the primary system of our catalogue.

The GC position and proper motion for each star were revised by incorporating the reduced position given in the LaPl30 essentially following the procedure suggested by Eichhorn and Googe (1969).

In the zone -47°30' to -52°, there were 176 stars common to the GC and Cp₅₀PP. In order to test for systematic discrepancies, the positions obtained from combining GC and LaPl30 were updated to the epoch of the Cape observation (roughly 1963). Working with the differences GC + LaPl30 minus Cp₅₀PP, an analysis was performed analogous to that done on the GC minus LaPl30 differences. The results yield the following formulas for reducing the Cp₅₀PP positions to our "GC on FK4" system in units of 0"001:

$$\Delta \propto \cos \theta = -192 - 175 \sin \alpha - 72 (m - 7\%5) - 96 (m - 7\%5) \sin \alpha$$
56 85 41 63

$$\Delta \delta = -200 \sin \alpha - 166 \cos 5\alpha - 189 \cos 3\alpha + 87 (m - 7^{-1}5) \sin \frac{1}{3} - 95 (m - 7^{-1}5) \cos 3\alpha.$$
78 74 80 52 59

(The small numbers under the coefficients are their standard errors.) The dispersion of the residuals from the solution which achieved the reduction was 0.50 in right ascension and 0.47 in declination. These corrections were then applied to all the Cp₅₀PP positions which were then used to revise the combined CC plus LaPl30, essentially in the same way LaPl30 was used to revise the CC.

For the final adjustment, all reference star positions were computed for the mean plate epoch using the proper motions derived from the combination

solution. No proper motions were applied to the $Cp_{50}PP$ positions for those stars not in the GC. After all, the epoch for any of these positions is at most a couple of years different from the mean plate epoch.

The reference positions in the overlap solution were in one of three categories and assigned standard errors as indicated: a. Coordinates combined from CC and LaPl30 (0.4), b. Coordinates combined from CC, LaPl30 and Cp50PP 3(0.119), and c. Cape positions (0.125), which means that the weights appropriate for cases a., b., and c., were 0.0625, 0.70 and 0.64, respectively under the assumption that a coordinate with a standard error of 0.11 would have weight 1.

V. THE UTILIZATION OF DIFFRACTION GRATING SPECTRA FOR ELIMINATING THE EFFECTS OF MAGNITUDE DEPENDENT ABERRATIONS

It is well known that magnitude dependent metric aberrations cause the stars' magnitudes to have an influence on the measured relative coordinates of their images' centers on the plates. This is acknowledged by writing the formulas for the computation of the standard coordinates ξ and η from the measured rectangular coordinates x and y in the form $\xi = \xi(x,y,m)$ and $\eta = \eta(x,y,m)$. Without restricting generality, this can be done by writing $\xi = \xi(x,y,m)$ and $\eta = \eta(x,y)$ with $\chi = \chi + \Delta \chi(x,y,m)$ and $\chi = \chi + \chi \chi(x,y,m)$, where χ and χ are the coordinates which the image would have if the star which generated it had been of the standard magnitude χ . This latter condition imposes the restriction that $\Delta \chi$ and Δy must be formulated such that $\Delta \chi(x,y,m_0) = 0 = \Delta y(x,y,m_0)$.

We now consider plates exposed with a coarse objective grating in front of the objective, such as were those from which the present catalogue was computed. We hypothesize that the ideal error- and aberration-free averages

 $\underline{x}_{\underline{d}} = \frac{1}{2}(\underline{x}_1 + \underline{x}_2)$, $\underline{y}_{\underline{d}} = \frac{1}{2}(\underline{y}_1 + \underline{y}_2)$ of the coordinates $(\underline{x}_1, \underline{y}_1)$ and $(\underline{x}_2, \underline{y}_2)$ of a pair of corresponding diffraction images would be equal to the coordinates \underline{x} , \underline{y} of the central image which they accompany. Then we have

 $\underline{x}_{\underline{d}} = \underline{x} + \Delta \underline{x}(\underline{x}, \underline{y}, \underline{m} + \mu), \ \underline{y}_{\underline{d}} = \underline{y} + \Delta \underline{y}(\underline{x}, \underline{y}, \underline{m} + \mu),$ where μ is the grating constant which is defined by the statement that a star of magnitude $\underline{m} + \mu$ would, on the same plate, have produced central images whose blackening is the same as that of a diffraction image produced by a star of magnitude \underline{m} . We have, in this work, assumed that μ is a constant for a given grating and a given order of diffraction spectra, that is, that μ varies neither with the position of the image on the plate, nor from one plate to the next. These assumptions appear justified from the theory of objective gratings (Bucerius, 1932).

We thus have, (neglecting the measurement errors),

$$\underline{\mathbf{x}} - \underline{\mathbf{x}}_{\mathbf{d}} = \Delta \underline{\mathbf{x}}(\underline{\mathbf{x}}, \underline{\mathbf{y}}, \underline{\mathbf{m}}) - \Delta \underline{\mathbf{x}}(\underline{\mathbf{x}}, \underline{\mathbf{y}}, \underline{\mathbf{m}} + \mu) \,, \underline{\mathbf{y}} - \underline{\mathbf{y}}_{\mathbf{d}} = \Delta \underline{\mathbf{y}}(\underline{\mathbf{x}}, \underline{\mathbf{y}}, \underline{\mathbf{m}}) - \Delta \underline{\mathbf{y}}(\underline{\mathbf{x}}, \underline{\mathbf{y}}, \underline{\mathbf{m}} + \mu) \,.$$

The left hand sides of these equations are linear functions of random variables, namely the measured coordinates, and are therefore random variables themselves. The right hand sides depend on the model functions chosen for Δx and Δy , which themselves depend only on quantities that may be regarded as known, namely (1) the measured coordinates, x and y, (2) the magnitudes of the individual stars (or their equivalents, such as the diameters of their images, etc.), and, (3) conceivably, the color indices (or their equivalents) of the stars. Color equivalents were, however, in this investigation, not available for all stars, and we have therefore made no attempt to look for, or remove, systematic effects on the positions which depend on color in any way.

Postulating the models

$$\Delta x = AM + BMx + CM^2 + DMx(x^2 + y^2)$$

$$\Delta y = EM+FMy+GM^2+HMy(x^2+y^2)$$

in which $\underline{M} = \underline{m}_{pg} - 8\underline{m}_{5}$ (where \underline{m}_{pg} was extracted from the CPC), we obtain

$$\underline{A}\mu + \underline{B}\mu \underline{x} + \underline{C}(\mu^2 + 2\underline{M}\mu) + \underline{D}\mu \underline{x}(\underline{x}^2 + \underline{y}^2) = \underline{x}_{\underline{d}} - \underline{x}$$

$$\underline{E}\mu + \underline{F}\mu \underline{y} + \underline{G}(\mu^2 + 2\underline{M}\mu) + \underline{H}\mu \underline{y}(\underline{x}^2 + \underline{y}^2) = \underline{y}_{\underline{d}} - \underline{y}$$

$$(3)$$

as equations of condition for the determinatin of the parameter \underline{A} , ..., \underline{H} by the method of least squares. The measurements on each pair of grating images yield one equation each in \underline{x} and \underline{y} . The differences on the right hand side of Eqs. (3) will (assuming that the measurements were made with a least count of one micrometer) be typically one- and two digit numbers, and we may therefore regard \underline{M} and the \underline{x} and \underline{y} occurring on the left hand side as known.

The parameter \underline{A} , ..., \underline{H} were determined by this method for each plate.

From the dispersion $\sigma_{\underline{d}}$ of the adjusted differences $\underline{x}_{\underline{d}} - \underline{x}$ and $\underline{y}_{\underline{d}} - \underline{y}$, the standard error $\sigma_{\underline{x}}$ of a coordinate measurement can be estimated by $\sigma_{\underline{x}} = \sqrt{2/3} \sigma_{\underline{z}}$. This estimate is of course a lower limit only because it was derived from the measurements made within a very small area of the plate and thus may be more characteristic for the setting error than for the true measurement error, which would, in addition to the setting error, be influenced by neglected errors introduced by the measuring machine or other components of the measuring process.

An analysis of the sets of parameters for each plate showed that the simpler models $\Delta x = AM+BMx$, $\Delta y = EM+BMy$ will adequately represent the magnitude dependent aberrations. In other words, we may assume come to affect the x and y coordinates equally. Since it was noticed that the dispersions of the values for B, as obtained for each plate, were close to, but somewhat significantly larger than, the averages of their standard errors as determined by the just described least squares solutions, we see that the individual values for B vary significantly from plate to plate. They do, however, cluster around a certain average value. In the final reductions this

was taken into account by adding a constraint to the equations of condition which encouraged the equality of the coma term B on overlapping plates, but did not enforce it rigorously. This follows in principle (but not in detail) the same procedure as applied by Eichhorn ad Gatewood (1967).3)

VI. THE FINAL ADJUSTMENT

In order to check for blunders, <u>preliminary plate constants</u> were computed using the reference stars in traditional adjustments in which each plate is treated independently from each other one. The adjustment model used for this was an affine transformation between measured and standard coordinates with the addition of linear magnitude terms and a linear coma term in each coordinate. The parameter estimates obtained in this adjustment were used to compute the coordinates of all stars and to compare positions obtained for the same stars from overlapping plates. Discrepancies of over 3.0 were investigated and rectified whenever their cause could be identified.

The coordinates published in this catalogue were computed using the plate overlap method as formulated by Googe et al. (1970). The principle underlying this approach is that all unknowns are to be calculated simultaneously from one large system of normal equations. The unknowns are, of course, the spherical coordinates of the stars and the (unpublished) plate parameters which are used to transform the measured coordinates to spherical ones.

The model postulated for the conversion of measured coordinates (x,y) to standard coordinates (ξ, η) was a complete cubic polynomial for both ξ and η . The coefficients of the terms of the cubic and those of the magnitude

³⁾A better, rigorous way to introduce these probabilistic constraints was developed by Eichhorn (1978) after the work on this catalogue was already finished.

equation were not subjected to any condition (other than those imposed by the algorithm itself). Since the details of the reduction closely follow those of similar work published previously (Lukac et al., 1970), we shall not repeat them here.

The standard error of a catalogued position estimate can be computed from the standard error of a position estimate which is based on a single image. This quantity was derived from the dispersion of the differences between the positions of the same star imaged on overlapping plates, and resulted as 0.17 in both $\alpha\cos\delta$ and δ . The standard error of a position based on the measurements of n images is therefore n.17/n. We should be careful, though, to note that measurements on diffraction images may not be as precise as those on central images, and that therefore those of our published catalogue positions which depend in part on diffraction images may be somewhat less precise than indicated by their formally calculated standard error.

VII. COMPARISON WITH THE FK4 SYSTEM

The positions in the present catalogue were compared with those in an unpublished catalogue on the FK4 system (Eichhorn, 1982) basically an update of the GC. The result of this comparison is given in Tab. V; the unit for declination corrections is 0.01; the corrections in right ascension are communicated in the form acos d and are in units of 0.001. A slight magnitude equation in right ascension was also found:

$\Delta \cos \delta = 0.0058 - 0.00143 m$

where <u>m</u> is the magnitude quoted in the catalogue. It must be noted that this magnitude equation should not be regarded as established for stars with a magnitude fainter than the faintest CC stars. It is, likewise, quite possible that the magnitude equation originated in the comparison catalogue rather than

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TABLE V.	

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2011		-18	2-	4		38	14	12
19h		-23	-12			25	12	G
181		-24	-17	တ		O	10	10
1 71			-21			9-	مد	2
16h			-12			9-	G	9
15h			4			10	16	15
14h			10			27	24	21
13h			9			29	24	22
1.2h		61	7	21		81	17	20
qe	80	61	14	21	Accoss	=	11	15
10h		97	18	20			13	
de de		19	14	15		19	11	21
48		14	œ	2			19	
42			2			15	15	. 22
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in our catalogue. The uncertainty of the table entries, which should in any case be regarded as preliminary, corresponds to a standard error of about 0.03 for the declination data and of 0.002 for those in right ascension in the form $\Delta s \cos \delta$.

VIII. DESCRIPTION OF THE CATALOGUE

A sample of data from the catalogue is enclosed as Tab. VI. The catalogue in its entirety has been deposited with the Physics Auxiliary Publication Service of the American Institute of Physics, see AIP document no. PAPS for 341 pages of tabular material of the same type as in Tab. VI.

The data have the following meaning: The first four columns identify the star by an already assigned number. 4)

The first and second columns give the number of the star in the CPD. Existing CPD numbers are, however, not always quoted. The second and third columns identify the star in one of the source catalogues, as follows: A indicates the Catalogue by Gill and Hough (1923; CpZo) and is followed by the star's number in this catalogue. B means that the number following is the star's number in the CPC zone -520 to -560. Other possible prefixes are the numbers from 1 through 360. The zones -49°, -51° and -53° of the AC, each covered by 120 plates, served as source catalogues for the faint stars; the numbers 1-120 refer to the plates in the AC zone -490, the numbers 121 to 240 to plates in the AC zone -510, and the plates in the AC zone -530 were assigned the numbers 241 through 360, always in order of increasing right ascension. The star carrying the designation 240 98 in our catalogue is thus star no. 98 on plate no. 120 in the AC zone -51°. This system allows one to specify plate number and zone by three digits without any additional information, which would not have been possible had we used the numbers for the plates which were assigned in the corresponding AC zones.

¹⁾ In view of the existing proliferation of star numbers we decided not to make matters even worse and thus refrained from assigning new star numbers for this

C.P.D. NUMBER	CATALOGUE SOURCE	STAR NUMBER	R.A. (1950)	DEC. (1950)	PHOTO-VISUAL MAGNITUDE	NUM	
	JOURCE	RSONUE			WAGNITUDE	CENT.	ε.
-5C 11889 -51 12055	1201	20784* 85 20785 10 88	23 55 11.72 23 55 12.81 23 55 13.57 23 55 15.38 23 55 16.81	-48 33 40.0 -48 33 10.0 -53 18 39.7 -48 33	8 • 8 13 • • 2 10 • 9 12	NNNN	2
-48 10988 -50 11890 -51 12056	120	98 20786 20787* 95 20789	23 55 23.00 23 55 26.01 23 55 49.85 23 55 49.87	-50 48 13.9 -48 23 27.0 -50 24 33.2 -48 23 28.5 -51 11 14.0	13.6 10.7 8.5 12.9 10.2	NNNN	2
-53 10561 -53 10562 -54 10412 -53 10564	120 8 8 8 8	100 9200* 9201 9204 9205	23 56 13.12 23 56 20:82 23 56 27.62 23 56 56.60 23 56 57.38	-49 15 16.9 -53 1 29.2 -52 47 13.7 -53 37 23.9 -53 7 18.7	12.7 10.1 11.1 e.1	2 2	2
-50 11891 -52 12236 -51 12063 -52 12237	1 2401	20758 9206 20800* 127 9207*	23 57 4.23 23 57 7.58 23 57 11.64 23 57 23.50 23 57 27.53	-50 6 38.8 -51 53 57.7 -51 16 38.5 -51 9 35.2 -52 10 19.4	10.7 9.6 7.8 11.6 8.3		2
-51 12065 -50 11892 -45 11836	120	118 20804* 122 20868 20812	23 57 28.66 23 57 34.32 23 57 41.08 23 57 42.69 23 57 47.94	-49 21 15.8 -51 8 45.4 -48 29 38.6 -49 37 51.1 -49 23 11.6	13.4 9.2 11.4 9.7 10.1	1 7 1	2
-53 10565 -53 10566	120 240 8 8 360	126 130 9209* 9210	23 57 49.35 23 57 49.41 23 58 .85 23 58 1.48	-48 52 7.1 -50 56 8.1 -53 22 33.3 -52 54 55.2 -53 32 4.5	13.7 11.2 6.9 9.7 11.1	12222	2
-51 12067	240 360 240 240	20815* 138 37 139 140	23 58 19.51 23 58 22.40 23 58 24.31 23 58 30.63	-50 43 30.2 -50 25 53.1 -52 41 9.7 -51 11 50.4 -51 23 33.0	8 · 3 12 · 2 11 · 4 13 · 9 12 · 2	2222	2
-49 1184C -53 10569 -53 1057C	240 120	20817* 141 9212 140 9213	23 58 30.81 23 58 30.85 23 58 33.67 23 58 39.91	-49 5 18.1 -50 23 10.7 -52 45 43.5 -48 17 4.8 -53 4 37.5	6.7 11.6 10.4 13.7 10.2	22212	2
-51 12068	240 240 120 120	20821* 147 145 142 143	23 58 46.96 23 58 46.96 23 58 58.83 23 58 58.83	-50 36 57.9 -50 26 58.6 -50 49 43.9 -48 30 54.7 -49 15 39.9	6.9 13.9 13.1 13.4 11.4	22222	2
-49 11843 -52 1224C	240	150 151 20825 155 9214	23 58 59.50 23 59 15.18 23 59 15.56 23 59 16.47 23 59 18.95	-51 2 5.7 -49 12 54.0 -48 50 27.8 -51 19 18.0 -52 32 28.4	13.4 13.2 9.6 11.9	NNNN	1 (

An asterisk indicates that the stars served as a reference star.

Columns 5 and 6 give right ascension and declination in their usual format, respectively, for the mean coordinate system 1950, ostensibly on the system of the FK4 and for the average epoch of the plates which provided data for the computation of the position. Should epochs be required with greater precision than the average epoch of all plates, they can be calculated in most cases from the data in Tab. I.

Column 7 gives the photovisual magnitudes, copied from existing sources. (For the AC stars, these were computed from the data listed in the AC.) These are neither very precise nor very accurate and should be used for identification purposes only.

Finally, columns 8 and 9 give the numbers of measured central images and pairs of diffraction spectra, respectively, that contributed to the calculation of the published position. The number of individual images measured for computing a published position is therefore the number in Col. 8 (2 in most cases) plus twice the number in Col. 9.

The catalogue is also available in the form of a magnetic tape which was distributed to the U.S. Naval Observatory in Washington, D.C., the NASA Data Center in Greenbelt, MD, The Centre des Données Stellaires in Strasbourg, France, the Astronomisches Rechen-Institut in Heidelberg, Germany and the Sydney Observatory in Australia. A copy also exists at the Department of Astronomy of the University of Florida (Gainesville, FL). Legitimate users may obtain copies and information about about the format under the customary conditions from any of these agencies.

One of us (H. E.) gratefully acknowledges a grant from the US Army Map Service which paid for the measurement of the plates. His colleague, Dr. John P. Oliver of the Department of Astronomy at the University of Florida gave generously of his time and expertise in matters concerning the preparation of the tape for distribution and of the printout of the catalogue for making the microfiche. The measuring team, then all students at the University of South Florida in Tampa, consisted of Messrs. J. Barrett, C. Dudley, R. Greenstone, R. Murphy, W. Scott, W. J. Veigel and Miss T. Schweitzer. The personnel who contributed substantially includes Messrs. D. Chambers, J. J. Draize, T. Hennig, A. Nickless, D. Oldham, D. Sloan, Msf. C. Leroy, and N. Lewis. The authors are profoundly grateful to these persons as well as to all their other collaborators, without whose conscientious efforts this catalogue could never have become a reality.

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